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The Lubrication of Rollers II. Film Thickness with Relation to Viscosity and Speed

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THE LUBRICATION OF ROLLERS

II. FILM THICKNESS WITH RELATION TO VISCOSITY AND SPEED

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CONTENTS

Systematic measurements with a disk machine of the thickness of the hydrodynamic oil film between loaded rollers have been made with respect to load, rolling speed, sliding speed and oil viscosity. It has been found that the viscosity of greatest importance with respect to film thickness is the viscosity of the oil at the surface temperature of the disks (η_s); the viscosity of the oil supplied by the lubricating jet and the viscosity attained by the oil on its passage through the pressure zone are unimportant.

It has been shown that film thickness is independent of load at loads exceeding 7×10^7 dyn cm^{-1}, but is dependent upon the rolling speed $\bar{u}$ (the mean peripheral speed of the disks) as well as upon η_s. To within ±15 % all the results are expressed by

$$h_D^* = 0\cdot8[(\bar{u}\eta_s)/100]^{0\cdot5},$$

where h_D^* is the film thickness in microns when $\bar{u}\eta_s$ is expressed in dyn cm^{-1}. Particularly is it noteworthy that this same expression remained true even when sliding was introduced. (The ranges of conditions covered by the experiments were loads from 7×10^7 to 2×10^8 dyn cm^{-1}, rolling speeds from 32 to 1000 cm s^{-1}, sliding speeds up to 480 cm s^{-1} and values of η_s from 0·14 to 1·19 P.) It is also shown that an implication of the insensitivity to sliding is that on the entry side there is little frictional heating of the oil due to the sliding up to a point where the pressure approaches 1×10^9 dyn cm^{-2}.

The experimental results have been compared with the theory of the elastohydrodynamic lubrication at the conjunction of the disks. The theory disregards frictional heating and predicts a film thickness proportional to $(\bar{u}\eta_s)^{0\cdot7}$ in contrast with the exponent of 0·5 given by experiment. Evidence is cited that the difference is not due to frictional heat but it is suggested that the discrepancy is due to a specific effect of speed upon the increase in viscosity which oils exhibit under pressure.

1. Introduction

In part I (Crook 1958) it was demonstrated that the lubrication of loaded rollers is hydrodynamic and a method of measuring the thickness of the hydrodynamic film was described. The measurements then made suggested that the film thickness is independent of load at loads greater than 7×10^7 dyn cm^{-1}, dependent upon rolling speed and the surface temperature of the disks, but insensitive to sliding. This paper describes systematic measurements of film thickness in relation to speed and viscosity. In particular from the various viscosities present in the system that of primary importance with respect to film thickness has been identified.

The apparatus (figure 1) comprised two rollers on parallel axes lubricated by a jet of oil and loaded together to be in nominal line contact as described in part I. The disks were sometimes run at unequal peripheral speeds and it was then convenient to regard the two speeds in terms of the rolling speed ($\bar{u}$) taken as the mean peripheral speed and the sliding speed ($u_2 - u_1$). The rolling speed is of importance with respect to the rate at which oil is dragged by the disks into the pressure zone at their conjunction while the sliding speed is of importance with respect to the frictional heating of the oil.

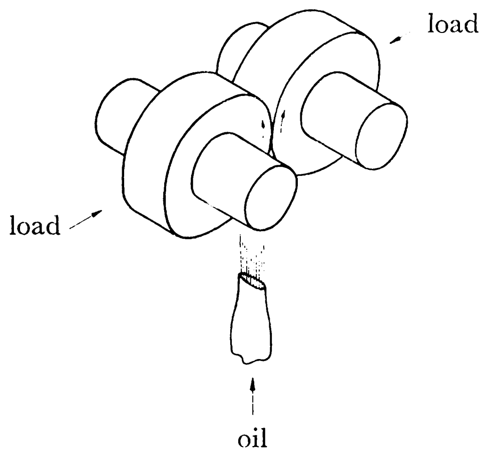


FIGURE 1. The disks.

In addition to the two speeds three viscosities have to be distinguished. There is the viscosity of the oil as supplied by the jet (η_{inlet}), the viscosity of the oil at the temperature of the surfaces of the disks but outside of the pressure zone (η_s), and the effective viscosity of the oil within the pressure zone ($\bar{\eta}_m$). Because the viscosities of oils increase under pressure $\bar{\eta}_m$ is in general much greater than either η_{inlet} or η_s. But the increase is mitigated by temperature so, as sliding is introduced, $\bar{\eta}_m$ falls as a result of the temperature rise resulting from the greater frictional heating. Because of this it was possible to vary $\bar{\eta}_m$ by introducing increasing amounts of sliding. (The measurement of $\bar{\eta}_m$ is a problem in itself and the values obtained have an intrinsic interest because they illumine the behaviour of oil under the peculiar physical conditions within the pressure zone. These matters will be discussed separately in part IV of this series of papers. The values of $\bar{\eta}_m$ quoted in this paper are taken from there.) The introduction of sliding, because of its production of greater frictional heating, also raised the temperature of the disks and therefore depressed η_s but to a lesser extent than it affected $\bar{\eta}_m$. Thus the introduction of sliding varied η_s and $\bar{\eta}_m$ in distinguishable ways and thereby permitted their influences upon film thickness to be separately assessed. The viscosity of the oil from the jet (η_{inlet}) was varied by changing the temperature of the supply.

In that way it has been established that the viscosity of prime importance with respect to film thickness is η_s. Variations of either η_{inlet} or of $\bar{\eta}_m$ are unimportant; a change in $\bar{\eta}_m$ by a factor of 40 changed the film thickness by only 10 %. The experiments also show that the film thickness is proportional to $(\bar{u}\eta_s)^{0\cdot5}$ both under conditions of pure rolling and of rolling with sliding; at constant $\bar{u}\eta_s$ the introduction of sliding has remarkably little effect upon film thickness.

Lastly in the paper the experimental results are compared with the theory as developed in approximate form by Grubin (1949) and more recently by Dowson & Higginson (1959, 1960) and Archard, Gair & Hirst (1961). The theory takes into account the effect of pressure upon viscosity and the elastic deformations of the surfaces resulting from the hydrodynamic pressures developed. It is fundamental to the theory that a value be given to the viscosity of the oil at each point throughout the film on the entry side and in doing this the viscosity to be attributed to the oil before it is affected by pressure is important. The present work has shown the relevant viscosity to be η_s. With that established, the film thicknesses predicted by theory are in reasonable accord with those given by experiment both under conditions of pure rolling and of rolling with sliding, despite the fact that the theory, because it does not embrace the additional complication of frictional heating, is isothermal. From this it has been deduced that the frictional heating due to sliding is small on the entry side up to a point where the pressure approaches 1×10^9 dyn cm^{-2}. (This deduction is confirmed in the following part III which is a theoretical discussion of frictional heating.) However, theory gives a film thickness proportional to $(\bar{u}\eta_s)$ raised to a power of approximately 0·7 whereas experiment gives a power of 0·5. Causes of experimental error have been scrutinized but none sufficient to account for the discrepancy was found. It is concluded that the experimental value of 0·5 is correct. A probable reason for the discrepancy is suggested.

2. Experimental

The disk machine described in part I was used. As before, film thickness was deduced from measurements of the electrical capacitance between the disks and flat steel pads riding upon the oil films carried away from the conjunction by the disks. Oil to Admiralty specification OM 100 was again used.

The viscosity of the oil adhering to the surfaces was obtained from the surface temperature and the known temperature dependence of viscosity at atmospheric pressure (figure 3 of part I). The surface temperatures were measured with fixed copper-constantan thermocouples bearing with a load of a few grammes on the moving tracks. The junctions were beaten to thin plates approximately 2 mm square. The e.m.fs were measured with a potentiometer.

With such thermocouples a number of effects including friction between junction and track might have caused errors.

The effect of friction was assessed by taking cooling curves on stopping the machine after it had been running for a time sufficient to establish equilibrium. The discontinuity shown by the curves on extrapolation back to the instant the machine stopped was less than 1 deg C (figure 2), so the error due to friction was therefore less than 1 deg C. The radiation and conduction errors were assessed by comparing the temperature recorded by the contact junction immediately after the machine had stopped with that recorded by a junction immersed in a small hole drilled close by the track of the disk. The difference of these temperatures was approximately 0·1 deg C. These results showed the contact junction to be accurate to approximately 1 deg C.

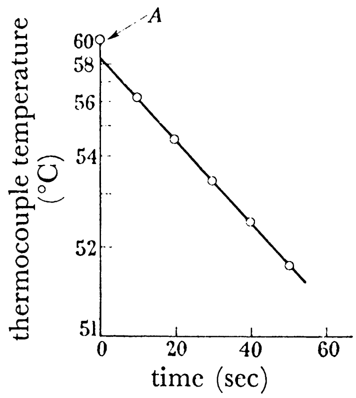


FIGURE 2. Cooling curve after stopping machine. (Point A indicates stable temperature with machine running.)

To discriminate between the effects of the temperatures of the inlet oil and of the surfaces it was essential to vary these temperatures independently. When there was no generation of heat within the disk machine the temperature of the whole machine was substantially that of the inlet oil. However, when sliding was introduced, friction between the disks produced heating which varied with the peripheral speeds and load; surface temperatures up to 40 deg C above the temperature of the inlet oil were obtained. This excess temperature could be controlled by varying the supply of inlet oil, thereby changing the degree of cooling. Control of the supply never starved the lubrication which requires only a very small feed of oil (part I).

In the experiments with rolling disks there was little frictional heat and there were never large differences between the temperatures of the inlet oil and of the surfaces. Such differences as did occur were due largely to the generation of heat at the bearings.

Film thicknesses were obtained over as wide a range as possible. The lower end of the range was limited by the development of electrical noise which ultimately prevented the measurement of capacitance. This noise arose from transient penetrations of the oil film. The upper end was limited by the maximum speed at which the machine could be run and by the surface temperature of the disks. Particularly with sliding a point was reached where the increase in thickness which would arise from an increment in speed was off-set by the effect of the concomitant rise in surface temperature.

In addition to the viscosities of the oil at the temperatures of the inlet jet and the surfaces of the disks, a knowledge of the effective viscosity within the conjunction of the disks ($\bar{\eta}_m$) was required. This latter viscosity is a subject of part IV and its measurement will not be described here.

3. RESULTS

Thicknesses (h^*_D) obtained over the range of experimental conditions given in table 1 are presented as functions of ($\bar{u}\eta_s$) in the logarithmic plots of figure 3. Figure 3 (a) refers to rolling conditions, while figures 3 (b) to 3 (d) each refer to experiments in which some sliding occurred. Each figure presents results in which the sliding speed (u_2-u_1) was a constant proportion of the rolling speed, i.e. of $\bar{u} = \frac{1}{2}(u_1+u_2)$.

In figure 4 the thicknesses of figure 3 (*b*) have been plotted as a function of the product of the rolling speed with the viscosity of the oil as supplied by the jet $(\bar{u}\eta_{\text{inlet}})$.

A feature of figure 3 is that h_D^* was approximately independent of load. In contrast, at lower loads (figure 5) the film thicknesses were dependent upon load.

TABLE 1. EXPERIMENTAL CONDITIONS

(1×10^7 dyn cm^{-1} = 57 Lb. in.$^{-1}$; 100 cm s^{-1} ~ 200 ft. min^{-1})

conditions	u_2/u_1	$(u_2-u_1)/\bar{u}$	range of load (10^7 dyn cm^{-1})	limits of $\bar{u}$ (cm s^{-1})	limits of (u_2-u_1) (cm s^{-1})	limits of η_s (P)
figure 3 (*a*)	1·0	0	7 to 20	50 to 1000	— —	0·37 to 1·19
figure 3 (*b*)	1·5	0·40	5 20	32 750	13 to 300	0·13 1·04
figure 3 (*c*)	2·0	0·67	7 20	150 600	100 400	0·16 0·56
figure 3 (*d*)	2·5	0·86	7 20	140 560	120 480	0·14 0·54

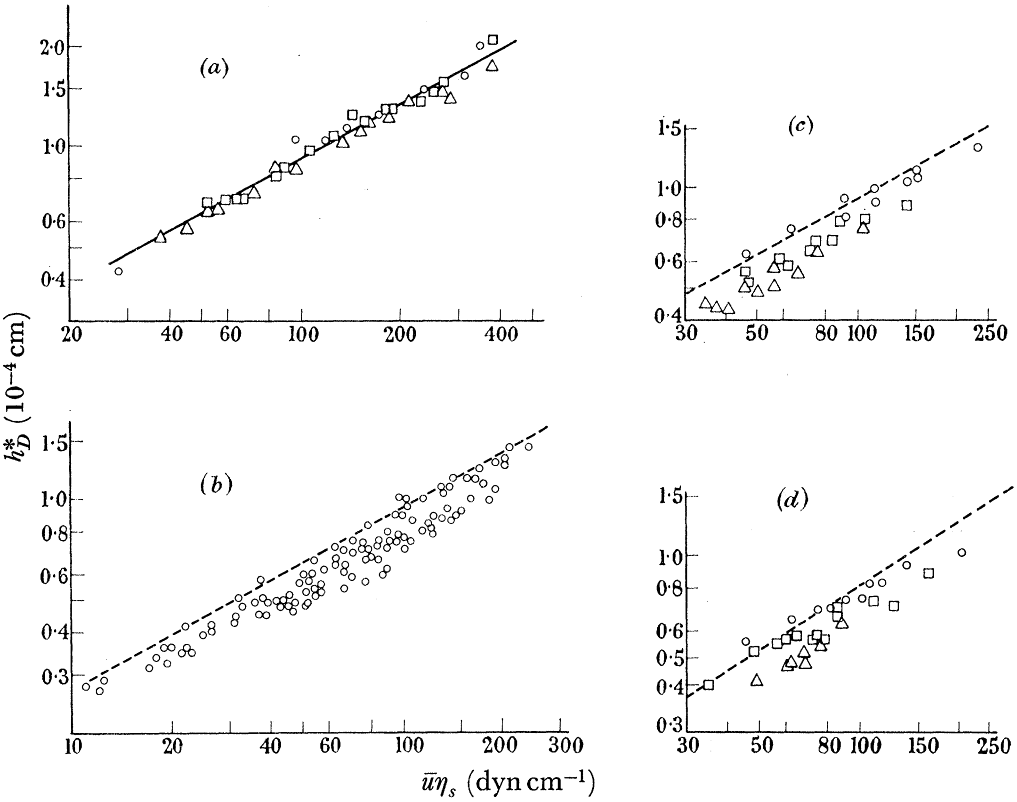


FIGURE 3. Film thickness as a function of $\bar{u}\eta_s$. (*a*) Rolling; (*b*) $u_2/u_1 = 1·5$; (*c*) $u_2/u_1 = 2·0$; (*d*) $u_2/u_1 = 2·5$. Load (*a*), (*c*) and (*d*): ○, $7·4\times10^7$ dyn cm^{-1}; △, $1·2\times10^8$ dyn cm^{-1}; □, 2×10^8 dyn cm^{-1}. In (*b*) loads from 5×10^7 to 2×10^8 dyn cm^{-1}; -----, line of (*a*) superimposed.

Some further experiments were conducted in which, unlike the previous experiments, the sum of the speeds of the disks was held constant as the sliding speed was varied and in which, by various artifices of technique, an attempt was also made to bring the surface temperature to the same constant value for each measurement. Film thickness and the effective viscosity within the pressure zone $(\bar{\eta}_m)$ were then measured. The results are presented in figure 6; $\bar{\eta}_m$ is shown as a function of sliding speed (logarithmic-linear plot) and the interrupted line shows h_D^* on a linear scale.

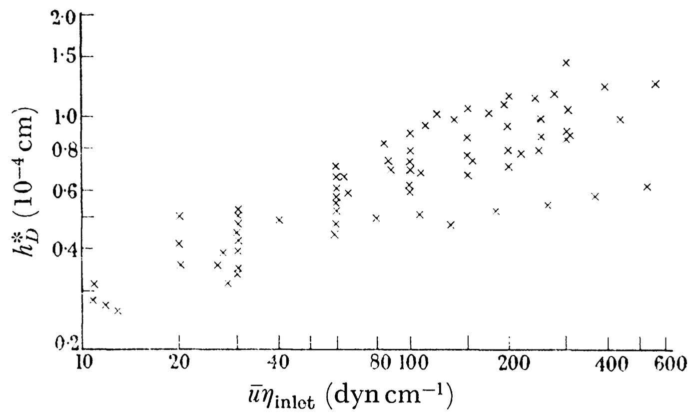


FIGURE 4. Film thickness as a function of $\bar{u}\eta_{\text{inlet}}$ (same experiments as for figure 3 (*b*)).

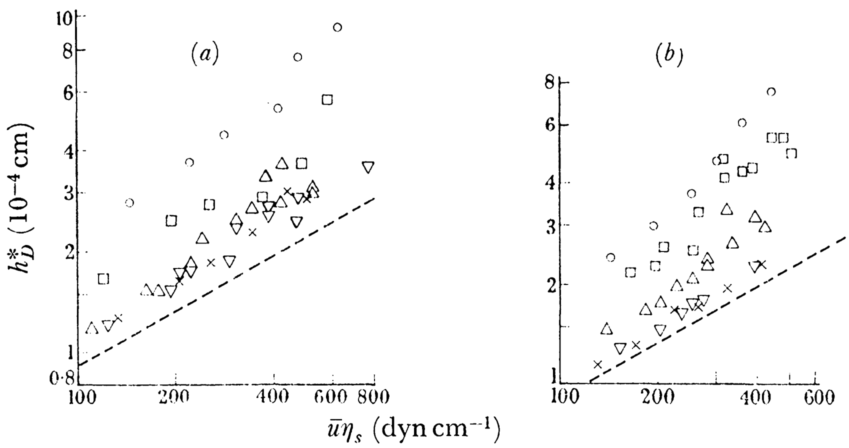


FIGURE 5. Film thickness at lower loads. (*a*) Rolling; (*b*) $u_2/u_1 = 1·5$. Loads: ○, $3·8 \times 10^6$ dyn cm^{-1}; □, $7·7 \times 10^6$ dyn cm^{-1}; △, $1·5 \times 10^7$ dyn cm^{-1}; ×, $2·3 \times 10^7$ dyn cm^{-1}; ▽, $3·1 \times 10^7$ dyn cm^{-1}; ––––, line of figure 3 (*a*).

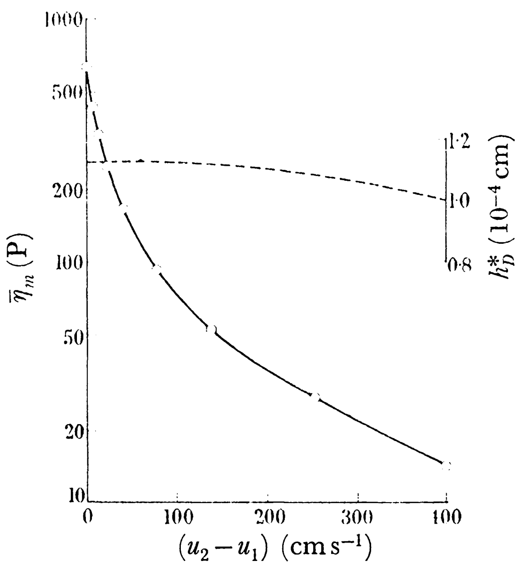


FIGURE 6. The effective oil viscosity within the pressure zone ($\bar{\eta}_m$) as a function of sliding speed. Load $7·4 \times 10^7$ dyn cm^{-1}; —○—, $\bar{\eta}_m$ (logarithmic scale); ––––, h_D^* (linear scale).

4. Discussion

(a) *The viscosity within the pressure zone*

Figure 6 shows that the effective viscosity within the pressure zone ($\bar{\eta}_m$) has only a small effect upon film thickness. There, as conditions changed from rolling to include 400 cm s^{-1} sliding, the value of $\bar{\eta}_m$ fell 40-fold from 600 to 14 P, yet despite that 40-fold drop, the value of h_D^* fell by only 10 %.

At the rolling point $\bar{\eta}_m$ was some thousand times greater than η_s and that is attributable to the effect of pressure upon the viscosity of the oil. An obvious cause of the 40-fold drop is that frictional heat was generated within the pressure zone upon the introduction of sliding. A similar drop must also have occurred between the experiments of figure 3 (*a*) (rolling) and those of figures 3 (*b*) to 3 (*d*) (with sliding). A comparison of the film thickness (the interrupted lines in figures 3 (*b*) to 3 (*d*) are the same as the solid line in figure 3 (*a*)) again shows that sliding caused little fall in film thickness.

It would appear, therefore, that the film thickness is determined by the amount of oil gathered into the pressure zone and is little influenced by the heat generated thereafter as the oil passes through that zone. This disregard of temperature has a simple explanation. Repeated integration of the Navier–Stokes equation (equation (4·2·1) of part I) leads to the expression

$$\eta = \frac{1}{12\bar{u}}\frac{\partial P}{\partial x}h^2\left(\frac{h}{h-h_D^*}\right), \tag{4·1}$$

where $\partial P/\partial x$ is the pressure gradient in the direction of motion of the disks, h and η are respectively the thickness and viscosity at x, and h_D^* is the thickness at the pressure maximum. From equation (4·1) it follows that the difference between h and h_D^* will be 10 % or less if

$$\eta \geqslant \frac{1}{12\bar{u}}\frac{\partial P}{\partial x}10h^2. \tag{4·2}$$

Let it now be assumed that the pressure distribution is Hertzian and that h is approximately 1 μ. Then if the conditions of figure 6 be taken, i.e. a load of $7·4 \times 10^7$ dyn cm^{-1} and a rolling speed of 800 cm s^{-1}, it can be shown that equation (4·2) becomes

$$\eta \geqslant 3·01 \cot\theta, \quad \cos\theta = x/b,$$

where b is the half-width of the Hertzian zone. The minimum value of η given by this expression is plotted as a function of x/b in figure 7 (logarithmic-linear plot). In addition an estimate of the lower limit of the actual viscosity available (the viscosity on the mid-plane of the oil film) when $(u_2 - u_1)$ was 400 cm s^{-1}, is given by the interrupted curve. (The method by which the estimate was made will be given in § 7 of part III.)

The figure shows that the actual viscosity available is more than sufficient to limit the variation of film thickness to 10 % of h (0·1 μ) over more than 80 % of the Hertzian band. This provides a justification for the original assumption of a Hertzian pressure distribution for it implies that the disks are flat through the zone. However, the assumption of a Hertzian distribution cannot be correct at the entry edge because some hydrodynamic pressures must develop beforehand. The important effect of these pressures is to reduce the value of $\partial P/\partial x$ where x approaches b, e.g. $\partial P/\partial x$ cannot be infinite, as in the Hertzian distribution, where x has the value b. Consequently, the figure exaggerates the minimum value of η required at the edge of the Hertzian flat and in fact the actual viscosity would exceed the minimum even closer to the edge than the figure suggests.

It should be stated that equation (4·2) does not take cognizance of the variation of η across the thickness of the film when, as in the context of the above argument, frictional heat is generated within the pressure zone. If this variation had been taken into account it would have complicated the calculation of the minimum viscosity required and would have led to slightly different values. The burden of the argument, that large variations in $\bar{\eta}_m$ can occur without upsetting the parallelism of the film, would, however, have been unchanged.

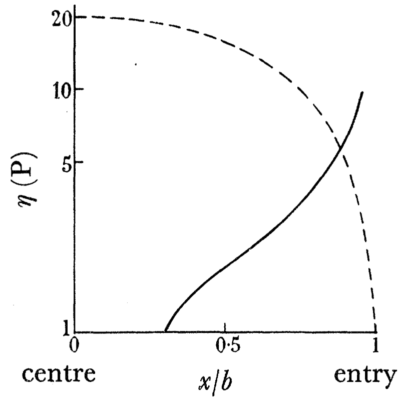


FIGURE 7. The value of η to limit variation of film thickness to 0·1 μ over Hertzian band (half-width b). ——, Minimum value of η required; ----, estimate of actual value of η on median plane of film. Data: load $7{\cdot}4\times10^7$ dyn cm^{-1}, $h_D^* = 1\ \mu$, $\bar{u} = 800$ cm s^{-1}, $u_2 - u_1 = 400$ cm s^{-1}.

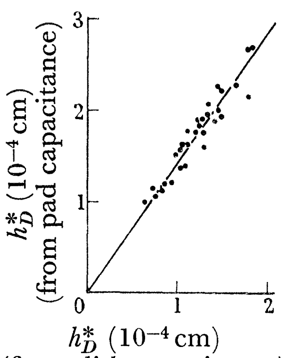


FIGURE 8. A comparison of thicknesses deduced from simultaneous measurements of disk and pad capacitance.

If the film be parallel then over the Hertzian band the two disks as separated by the oil film form a parallel plate condenser and, after allowance has been made for the capacitances due to the entry and recess zones, a film thickness can be deduced from measurements of the total capacitance between the disks themselves. In figure 8 film thicknesses as deduced from measurements of pad capacitance are compared with those deduced from simultaneous measurements of disk capacitance. The range of load was from $7{\cdot}4\times10^7$ dyn cm^{-1} to 2×10^8 dyn cm^{-1}, the range of $\bar{u}\eta_s$ was from 50 to 1200 dyn cm^{-1} and in some instances the ratio of the peripheral speeds was 1:1·5. For reasons of a detailed nature the two sets of thicknesses are not exactly equal. But they are proportional, which confirms that the relative thicknesses deduced from pad capacitances are accurate over a wide range of $\bar{u}\eta_s$ and also confirming that the film between the disks is effectively of constant thickness over the Hertzian band.

(b) *The important viscosity*

In general, theories of hydrodynamic lubrication lead to expressions in which speed and viscosity are associated as a product and indeed dimensional analysis indicates that should be so.

It has been shown above, from experiments in which $\bar{u}$ was held constant, that $\bar{\eta}_m$ had little influence upon thickness. In contrast, figures 3 and 4 show that correlations exist between h_D^* and $(\bar{u}\eta_s)$ and between h_D^* and $(\bar{u}\eta_{\text{inlet}})$. However, a comparison of figure 4 with figure 3 (*b*) shows that the correlation with $(\bar{u}\eta_s)$ is the stronger.

Table 2. Comparison of correlation coefficients

(Same experimental results as for figure 3.)

(The probability is that of the difference between r_1 and r_2 being exceeded by chance.)

ratio of peripheral speeds	correlation of log h_D^* with log $\bar{u}\eta$ suffix 1, $\bar{u}\eta_s$; suffix 2, $\bar{u}\eta_{\text{inlet}}$		
	r_1	r_2	probability
1:1	0·988	0·965	5×10^{-3}
1:1·5	0·941	0·334	1×10^{-12}
1:2	0·950	0·447	2×10^{-7}
1:2·5	0·945	0·495	3×10^{-6}

This was established more definitely by calculating correlation coefficients. The results are given in table 2. The coefficients (r) relate actually to the logarithms of h_D^* and $(\bar{u}\eta)$. The last column shows the probability calculated via Fisher's z transformation (Fisher 1954) that a difference exceeding the difference between r_1 and r_2 should arise by chance. The probabilities are small and even the difference between the correlations from results with rolling disks is considered to be significant. Thus, in all instances the correlation with $(\bar{u}\eta_s)$ was significantly better than the correlation with $(\bar{u}\eta_{\text{inlet}})$.

That result is consistent with previous observations. It has been shown experimentally that the oil films established upon the disks suffice for lubrication even after the supply of oil from the jet has been cut off (part I) and, furthermore, that extraneous oil does not merge readily with these films (Crook 1957). This suggests that the oil actually responsible for the lubrication is carried by the disks as skins. Such skins would be in thermal equilibrium with the disks rather than with the inlet oil which does not merge with them. From that a causal connexion between $(\bar{u}\eta_s)$ and thickness is therefore to be expected.

The surface temperature, however, can never be uninfluenced by the inlet temperature since this mainly sets the ambient temperature of the machine. Thus, even though the causal connexion be with $(\bar{u}\eta_s)$, a derivative correlation with $(\bar{u}\eta_{\text{inlet}})$ is nevertheless to be expected.

(c) *The relation between h_D^* and $(\bar{u}\eta_s)$*

Figure 3 shows that a power relation exists between h_D^* and $(\bar{u}\eta_s)$. It is convenient to express this relation as

$$h_D^* = h_m[(\bar{u}\eta_s)/100]^n, \tag{4·3}$$

where h_m is the value assumed by h_D^* when $(\bar{u}\eta_s)$ is 100 dyn cm^{-1}. That value of $(\bar{u}\eta_s)$ has been chosen as it is approximately the geometric mean of the range explored. To within ± 15 % all the experimental results at loads greater than 7×10^7 dyn cm^{-1} are expressed by

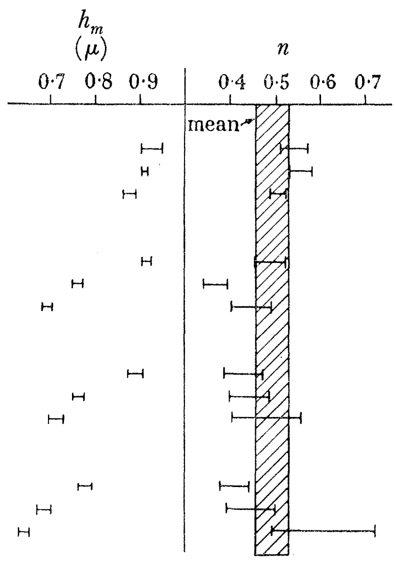
$$h_D^* = 0{\cdot}8[(\bar{u}\eta_s)/100]^{0{\cdot}5}, \tag{4·4}$$

where h_D^* is in microns. (In figure 9 of part I a thickness as measured in pure rolling proportional to $(\bar{u})^{\frac{1}{3}}$ was shown. However, the importance of η_s was not fully recognized.) The experimental results were analyzed statistically to assess the influence of sliding and load upon n and h_m. For each set of results the regression of $\log h_D^*$ upon $\log(\bar{u}\eta_s)$ was calculated. (The relative values of h_D^* are correct to within a few parts per cent; the absolute values may have a larger constant error, perhaps up to 30 %.) The regression coefficient gave the value of n. The value of h_m was taken as the value of h_D^* given by the regression line when $\bar{u}\eta_s$ was 100 dyn cm^{-1}. From the variances and a table of the 't' distribution 90 % confidence limits were found. The results of the analysis are given in table 3.

Table 3. Statistical analysis of results

(Bars indicate 90 % confidence limits.)

u_2/u_1	load (10^7 dyn cm^{-1})
1·0	7·4
	12·0
	20·0
1·5	7·4
	12·0
	20·0
2·0	7·4
	12·0
	20·0
2·5	7·4
	12·0
	20·0



They show that upon the introduction of sliding there was a tendency for n to be depressed but no consistent variation of n with load was detected. But there was a marked fall in h_m with increasing load which did not occur to the same degree under rolling conditions. Nevertheless, the change in h_m with load when the disks rolled was significant; the probability of it being exceeded by chance was only 1 %. But even though the changes in h_m were significant and particularly so with sliding, the change between the extremes of rolling at the lowest load and of the highest load combined with the most severe sliding was only 30 %. When the magnitude of the corresponding variations in $\bar{\eta}_m$ is recalled the important fact is not that significant variations in h_m occurred but that they were no greater.

(d) *Loads less than* $7{\cdot}4 \times 10^7$ dyn cm^{-1}

Figures 5 (*a*) and (*b*) show that at low loads film thicknesses when measured at constant $\bar{u}\eta_s$ were also independent of sliding. However, as found in part I, they were not independent of load.

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(e) *The comparison of experiment with theory*

The experimental results show that film thickness is insensitive to sliding. The implication of this insensitivity may be seen by considering it together with Grubin's (1949) approximate theory of film thickness. He has shown that if viscosity varies with pressure according to

$$\eta_p = \eta_0 \exp \delta P \tag{4.5}$$

then equation (4·1) leads to

$$P' = 12\bar{u}\eta_0 \int_x^\infty \left(\frac{h-h_D^*}{h^3}\right) \mathrm{d}x, \tag{4.6}$$

where

$$P = -(1/\delta) \ln (1-\delta P'); \tag{4.7}$$

P is the real pressure to which P' is related by the pressure coefficient of viscosity δ. The maximum possible value of P' is $1/\delta$ for then P becomes infinite.

The integral of equation (4·6) reaches its full value once h falls to h_D^* which, as already argued, occurs at the entry edge of the Hertzian band. (Here the entry edge will be taken as the origin ($x = 0$).) Grubin assumed that the shape of the disks is everywhere Hertzian and by numerical integration of a number of examples expressed the integral as a function of h_D^*. More simply, however, it can be shown that the Hertzian shape is expressible in approximate form as

$$h - h_D^* = h_D^* \chi^{\frac{3}{2}}, \quad \chi = 2b^{\frac{1}{3}}x/(3Rh_D^*)^{\frac{2}{3}}, \tag{4.8}$$

where R is the relative radius of curvature of the disks and b is the half-width of the Hertzian band; with a single numerical integration it is then found that

$$h_D^* \approx 2{\cdot}4(\bar{u}\eta_0)^{\frac{3}{4}} R^{\frac{1}{2}}/b^{\frac{1}{4}}P'^{\frac{3}{4}}. \tag{4.9}$$

The maximum value of P' is $1/\delta$ which corresponds to an infinite real pressure. But a real pressure of 1×10^9 dyn cm^{-2}, a fraction of the maximum Hertzian pressure at the loads being considered (e.g. 6×10^9 dyn cm^{-2} at a load of 2×10^8 dyn cm^{-1}), gives a P' of only 10 % less, i.e. $0{\cdot}9/\delta$ (δ taken as $1{\cdot}6\times10^{-9}$ dyn^{-1} cm^2). Thus, if a pressure of 1×10^9 dyn cm^{-2} precedes the entry edge there P' may be set at $1/\delta$ with the possibility arising thereby of an error in h_D^* of only 8 % (equation (4·9)). But of course the theory is only valid provided η varies with pressure approximately in accord with equation (4·5) up to a pressure of 1×10^9 dyn cm^{-2} and that would not be true if frictional heat depressed the viscosity before a pressure of 1×10^9 dyn cm^{-2} was reached. Therefore an implication of the insensitivity of film thickness to sliding is that there is little frictional heat on the entry side up to pressures approaching 1×10^9 dyn cm^{-2}. (It also follows from the theory that when it is permissible to set P' equal to $1/\delta$ at the entry edge then film thickness is insensitive to load. But at low loads that cannot be done and then, as in figure 5, the film thickness depends markedly upon load.)

When cast in the same form as equation (4·3) Grubin's theoretical result is

$$h_D^* = h_m[(\bar{u}\eta_s)/100]^{0{\cdot}727},$$

where h_m is slightly dependent upon load, e.g. $0{\cdot}66\,\mu$ at a load of $7{\cdot}4\times10^7$ dyn cm^{-1} and $0{\cdot}60\,\mu$ at a load of 2×10^8 dyn cm^{-1}; equation (4·9) gives an exponent of 0·75 and corresponding values of h_m of $0{\cdot}79\,\mu$ and $0{\cdot}70\,\mu$. The experimental value of h_m was $0{\cdot}8\,\mu$, so at a

$\bar{u}\eta_s$ of 100 dyn cm^{-1} experiment and theory agree reasonably. But the experimental exponent was 0·5, significantly less than Grubin's. Recently with digital computers more exact calculations have been made (Dowson & Higginson 1959, 1960; Archard *et al.* 1961). These also give exponents of approximately 0·7. So in respect of the exponent there is a discrepancy between experiment and theory. Because of this it became necessary to examine with particular care whether the nominal experimental exponent of 0·5 had been influenced by errors of measurement varying systematically with $\bar{u}\eta_s$. Subsidiary experiments conducted in that connexion are described in the appendix. No errors approaching the required magnitude were found.

This discrepancy between the exponents as found experimentally and as predicted by theory brings into question the assumptions of the theory concerning the entry side in their finer detail and in particular the neglect in the theory of frictional heating. In part III a theoretical analysis of frictional heating is given. It confirms that the heating due to sliding on the entry side is small up to a pressure of 1×10^9 dyn cm^{-2} and thereby explains the insensitivity of film thickness to sliding. But in addition it shows that the frictional heating due to the rolling motion of the disks is also small; too small to account for the discrepancy. Thus attention becomes directed to other assumptions.

With respect to the variation of viscosity with pressure the theory rests upon high pressure measurements made with apparatus following Bridgman in which the pressure is continuously maintained. The values so obtained may not be accurate under the transient pressures experienced by the oil in its passage through the conjunction. In part IV an account will be given of measurements of the actual effective viscosity of the oil in its passage through the conjunction. The results show that the increase in viscosity with pressure lessens as the rolling speed is increased. This can be regarded as a reduction in the pressure coefficient of viscosity (δ of equation (4·5)) as the speed increases; which, as may be seen from equation (4·9) and the argument which follows it, would lead to a less rapid rise in film thickness with speed than would be found were δ constant. The evidence is not decisive but at present the effect appears to be the most probable cause of the discrepancy.

5. Conclusion

For the range of conditions explored the experiments show that the viscosity of greatest importance with respect to film thickness is the viscosity of the oil at the surface temperature of the disks. The fact that η_s rather than η_{inlet} is important supports the suggestion (Crook 1957) that the oil responsible for the lubrication is carried by the surfaces as skins in thermal equilibrium with the disks. It has been shown that there is no considerable reduction in film thickness upon the introduction of sliding despite the accompanying increase in frictional heating and the consequent reduction in $\bar{\eta}_m$. From that it has been deduced that there is little frictional heating on the entry side up to a point where the pressure approaches 1×10^9 dyn cm^{-2}.

It has been shown that the magnitude of the thickness as found experimentally is comparable with that predicted by theory. But experiment shows film thickness to be proportional to $(\bar{u}\eta_s)$ raised to a power of approximately 0·5, whereas theory predicts an exponent of approximately 0·7. The experimental errors are insufficient to account for the discrepancy and it is concluded that the experimental exponent of approximately 0·5 is correct. The explanation of why the experimental exponent lies beneath that predicted by theory has yet to be determined, but there is evidence that the explanation lies in a reduction of the pressure coefficient of viscosity as the rolling speed is increased.

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Appendix. The accuracy of the results

The experiments show that film thickness is proportional to $(\bar{u}\eta_s)^{0\cdot5}$ whereas theory predicts an exponent of approximately 0·7. But before the exponent of 0·5 can be accepted the possibility of experimental errors depressing an exponent of 0·7 to a value of 0·5 has to be discounted. In this appendix the experimental errors are scrutinized.

The error in $\bar{u}$ was small and can be discounted immediately. An error in η_s would arise if the measured values of surface temperature (θ_s) were incorrect. But it can be shown that if the variation of η with temperature be taken as

$$\eta_\theta = \eta_0 \exp(-\gamma\theta),$$

then for each decade by which $\bar{u}\eta_s$ increases the measured values of θ_s would have to be progressively too small by $0\cdot3/\gamma$ to account for the discrepancy. If γ be taken as 2×10^{-2} deg C^{-1} this gives an error in θ_s of 15 deg C for each decade by which $\bar{u}\eta_s$ increases. In § 2 it has been shown that the total error in θ_s was of the order 1 deg C. Consequently the discrepancy between experiment and theory cannot arise from errors in the measurement of surface temperature.

The possibility that the discrepancy arises from errors in the measurement of film thickness will now be considered. The thicknesses (h_D^*) quoted in the results were derived from the measured pad capacitance (C_F) by use of the relation (part I)

$$C_F = \beta h_D^{*-0\cdot5}.$$

The value of β depends upon the degree to which the space between a pad and its disk is filled with oil (appendix of part I) If the oil filling were to vary with $\bar{u}\eta_s$ there would be a variable error in the values of h_D^* delivered by the above relationship when a constant β is assumed. It can be shown that to account for the discrepancy in these terms β would have to fall by 26 % for each decade by which $\bar{u}\eta_s$ increases.

The method described in the appendix of part I was used to examine the actual variation of β over a range of $\bar{u}\eta_s$ from 20 to 450 dyn cm^{-1}. All the values of β lay within ± 2 % of the mean; there was no variation approaching the 26 % per decade of $\bar{u}\eta_s$ required to account for the discrepancy in terms of a variable value of β.

The comparison of film thickness as deduced from simultaneous measurements of pad and disk capacitances (figure 8) also confirms that the relative thicknesses as deduced from the pad capacitances are accurate.

Errors in $\bar{u}$, η_s and h_D^* have been considered. None sufficient to account for the discrepancy between experiment and theory have been found and it is concluded that the experimental exponent of 0·5 is correct.

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